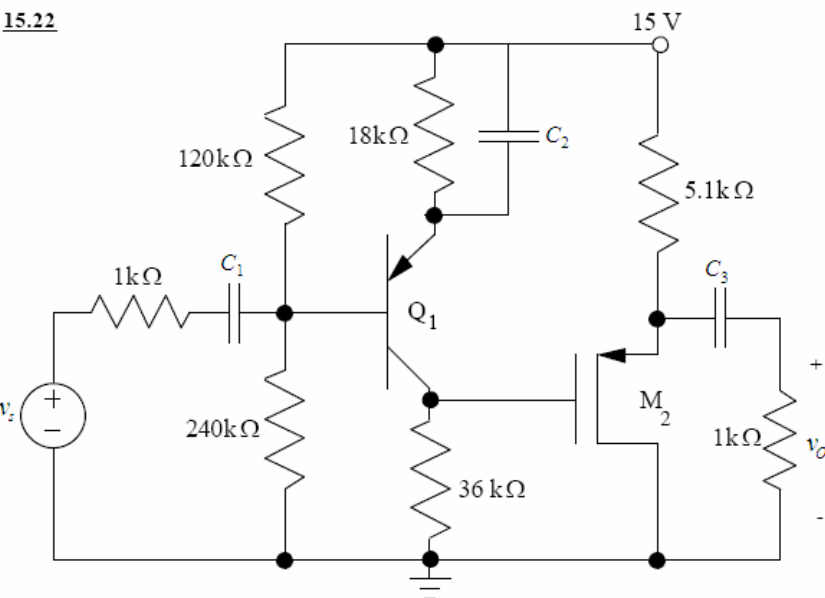
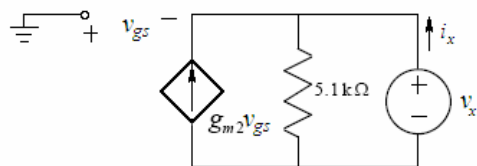
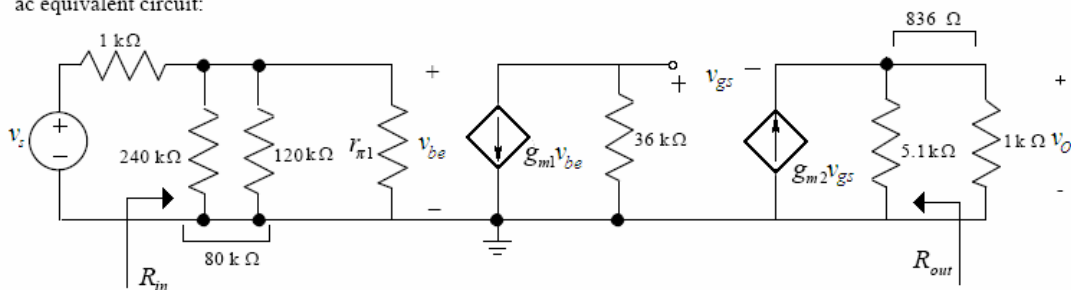


15.22



ac equivalent circuit:



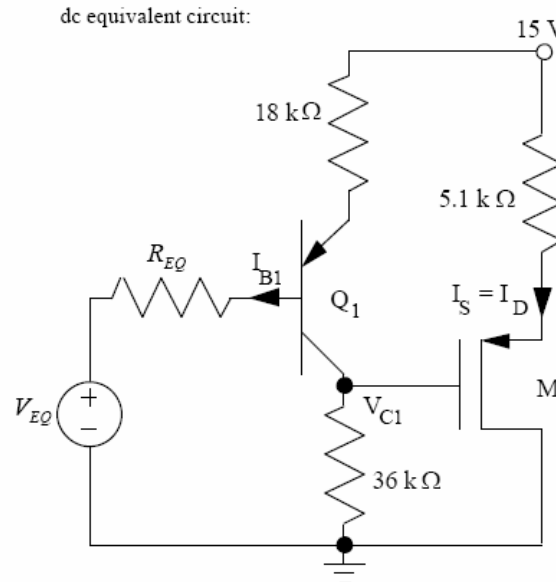
$$i_x = \frac{v_x}{5100} - g_{m2}v_{gs}$$

$$i_x = \frac{v_x}{5100} - g_{m2}(-v_x)$$

$$i_x = v_x \frac{1 + (g_{m2} 5100)}{5100}$$

$$R_{out} = \frac{v_x}{i_x} = 5100 \parallel \frac{1}{g_{m2}} = 241 \Omega$$

dc equivalent circuit:



$$V_{EQ} = \frac{240k\Omega}{240k\Omega + 120k\Omega} 15V = 10.0V$$

$$R_{EQ} = 240k\Omega \parallel 120k\Omega = 80.0k\Omega$$

Assume Depletion Mode PMOS Operates in Saturation and BJT Operates in Forward Active Region;

$$I_{B1} = \frac{15 - 10 - 0.7V}{80 + 76(18)k\Omega} = 2.97 \mu A \quad | \quad I_{C1} = 75I_{B1} = 223 \mu A$$

$$V_{E1} = 15 - 18000I_{E1} = 10.9V \quad | \quad V_{C1} = 36000I_{C1} = 8.03V$$

$$V_{EC1} = 10.9 - 8.03 = 2.87V \quad | \quad V_{CB} = 0.7 - 2.87 = -2.17V \quad \text{Assumption is Correct}$$

$$I_{D2} = \frac{15 + V_{GS2} - 8.03}{5.1k\Omega} \quad | \quad I_{D2} = \frac{0.004}{2}(V_{GS2} - 4)^2 \Rightarrow V_{GS2} = 3.01V, \quad I_{D2} = 1.96mA$$

$$V_{DS2} = -[15 - 5.1k\Omega(1.96mA)] = -5.00V \quad | \quad V_{GS} - V_p = 3.01 - 4 = -0.99V$$

$$|V_{DS2}| > |V_{GS} - V_p| \quad \text{Assumption is Correct}$$

$$g_{m1} = 40(223\mu A) = 8.92mS \quad | \quad r_{\pi1} = \frac{75}{8.92mS} = 8.41k\Omega$$

$$g_{m2} = \sqrt{2(4 \times 10^{-3})(1.96 \times 10^{-3})} = 3.96mS$$

$$R_m = 240k\Omega \parallel 120k\Omega \parallel r_{\pi1} = 7.61k\Omega \quad | \quad R_L = 5.1k\Omega \parallel 1k\Omega = 836\Omega$$

$$A_{v1} = \frac{v_g}{v_i} = -g_{m1}(36k\Omega) \left(\frac{R_m}{R_L + R_m} \right) = -8.92mS(36k\Omega) \frac{7.61k\Omega}{1k\Omega + 7.61k\Omega} = -284$$

$$A_{v2} = \frac{v_o}{v_g} = \frac{g_{m2}R_L}{1 + g_{m2}R_L} = \frac{(3.96mS)(0.836k\Omega)}{1 + (3.96mS)(0.836k\Omega)} = 0.768$$

$$A_v = 0.768(-284) = -218$$

(b) If the bypass capacitor is removed from the 18 kΩ resistor, then

$$A_{v1} = \frac{v_g}{v_i} = \left(\frac{R_{in}}{1k\Omega + R_{in}} \right) \left(\frac{-\beta_{ol}R_f}{r_{\pi1} + (\beta_{ol} + 1)18k\Omega} \right)$$

$$A_{v1} = \left(\frac{75.6k\Omega}{1k\Omega + 75.6k\Omega} \right) \left(-\frac{75(36k\Omega)}{138k\Omega} \right) = -1.93$$

$$A_v = 0.768(-1.93) = -1.48$$

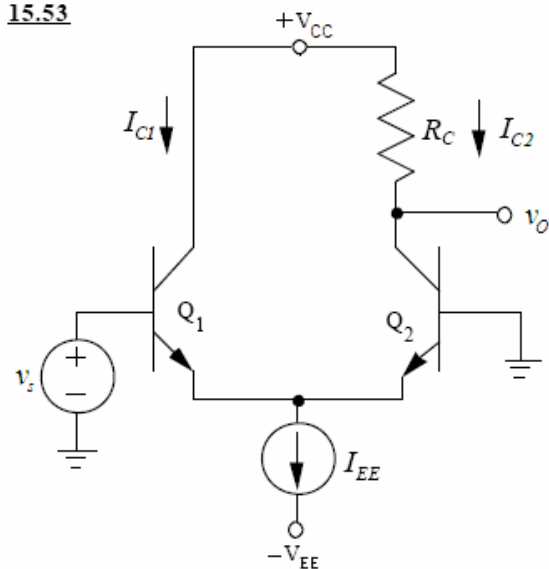
$$i_x = \frac{v_x}{80k\Omega} + \frac{v_x}{[r_{\pi1} + (\beta_{ol} + 1)18k\Omega]}$$

$$i_x = v_x \frac{[r_{\pi1} + (\beta_{ol} + 1)18k\Omega] + 80k\Omega}{80k\Omega[r_{\pi1} + (\beta_{ol} + 1)18k\Omega]}$$

$$R_{in} = \frac{v_x}{i_x} = 80k\Omega \parallel [r_{\pi1} + (\beta_{ol} + 1)18k\Omega]$$

$$R_{in} = 75.6k\Omega$$

15.53



Assume Both BJT Operate in Forward Active Region ;

$$I_{E1} = I_{E2} = \frac{I_{EE}}{2} \left(I_{E1} = I_{E2} = I_S e^{\left(\frac{V_{BE}}{V_T} \right)} \quad V_{BE1} = V_{BE2} = V_{BE} \right)$$

$$I_{C1} = I_{C2} = I_C = \alpha_F I_E = \alpha_F \frac{I_{EE}}{2} = \frac{100}{101} \left(\frac{20 \mu A}{2} \right) = 9.90 \mu A$$

$$V_{C2} = 10 - 9.1 \times 10^5 I_C = 0.991 V$$

$$g_{m1} = g_{m2} = g_m = 40 I_C = 0.396 mS \quad r_{\pi 1} = r_{\pi 2} = r_{\pi} = \frac{\beta_o}{g_m} = \frac{100}{0.396 mS} = 252.5 k\Omega$$

(a) For $v_s = 0$:

$$V_O = V_{C2} = 0.991$$

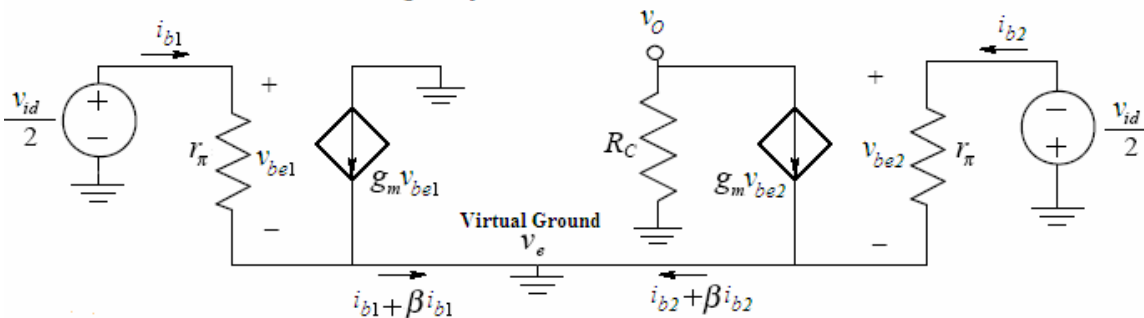
$v_o = 0$ since there is no independent source

$$v_o = v_o + V_O = 0.991$$

For $v_s = 2mV$:

$$V_O = V_{C2} = 0.991$$

Differential mode input : $v_{id} = v_1 - v_2 = v_s - 0 = 2mV$



$$i_{b1} + \beta i_{b1} = -(i_{b2} + \beta i_{b2})$$

$$i_{b1} = -i_{b2}$$

$$v_{id} = i_{b1} r_{\pi} - i_{b2} r_{\pi}$$

$$v_{id} = -2 i_{b2} r_{\pi}$$

$$i_{b2} = -\frac{v_{id}}{2 r_{\pi}}$$

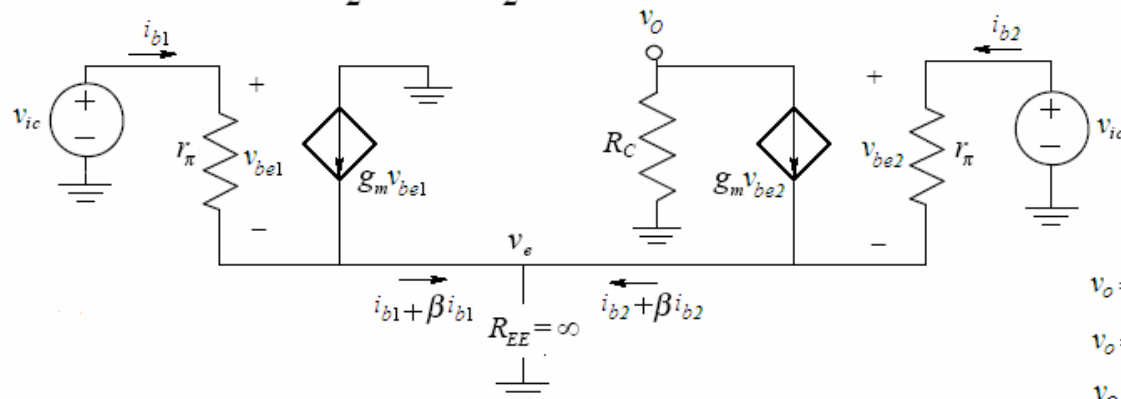
$$v_o = -g_m v_{be2} R_C$$

$$v_o = -g_m i_{b2} r_{\pi} R_C$$

$$v_o = -g_m \left(-\frac{v_{id}}{2 r_{\pi}} \right) r_{\pi} R_C$$

$$\text{Ended Output} \rightarrow A_{dd} = \frac{v_o}{v_{id}} = \frac{g_m R_C}{2} = 180$$

Common mode input : $v_{ic} = \frac{v_1 + v_2}{2} = \frac{2mV + 0}{2} = 1mV$



$$A_{cc} = \frac{v_o}{v_{ic}} = \frac{0}{v_{ic}} = 0 \quad \text{Since there is no current}$$

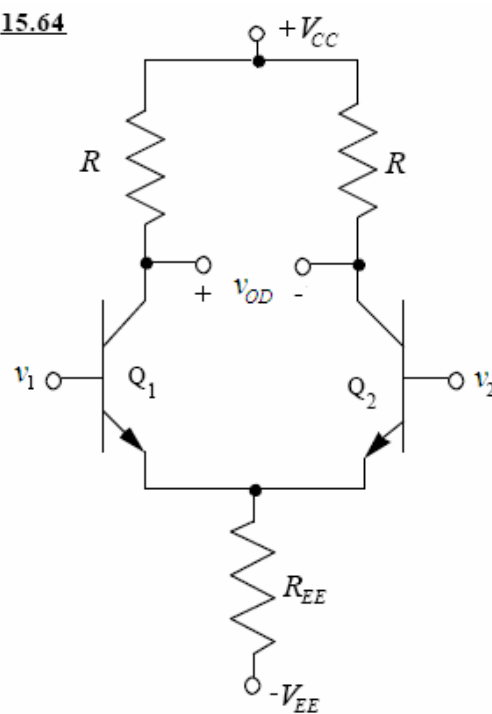
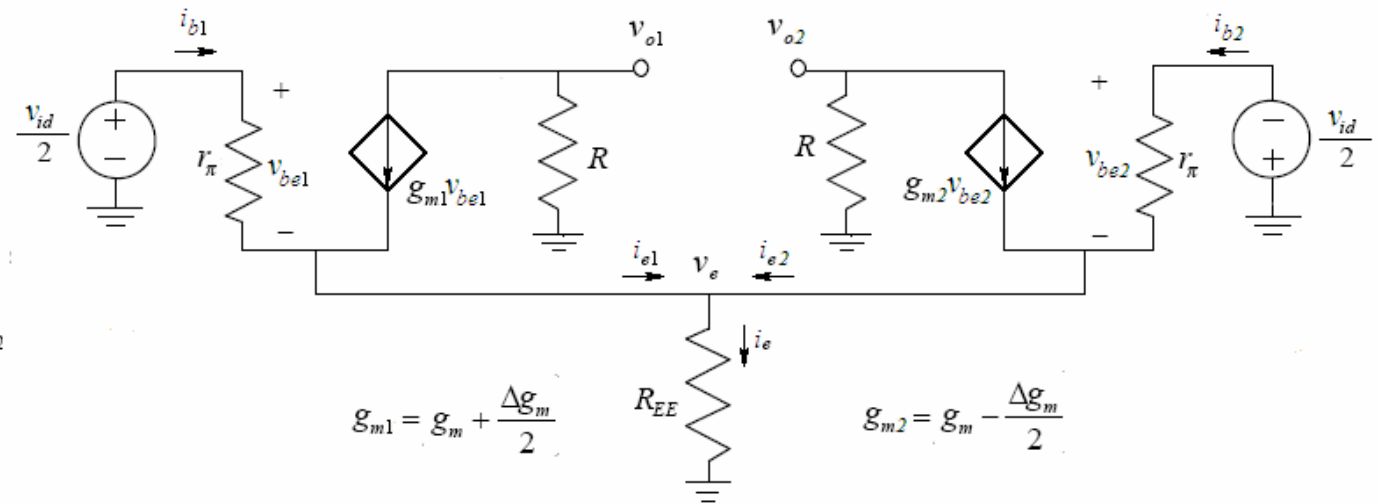
corrected

$$v_o = A_{dd} v_{id} + A_{cc} v_{ic}$$

$$v_o = 180(0.002V) - 0(0.001V) = 0.36V$$

$$v_o = v_o + V_O = 0.36V + 0.991V = 1.35V$$

15.64

Differential mode input : v_{id} 

There is no virtual ground since the circuit is mismatched

$$v_{be1} = \frac{v_{id}}{2} - v_e$$

$$v_{be2} = -\frac{v_{id}}{2} - v_e$$

$$g_{m1} v_{be1} = g_{m1} \left(\frac{v_{id}}{2} - v_e \right)$$

$$g_{m2} v_{be2} = g_{m2} \left(-\frac{v_{id}}{2} - v_e \right)$$

$$v_{o1} = -g_{m1} \left(\frac{v_{id}}{2} - v_e \right) R$$

$$v_{o2} = -g_{m2} \left(-\frac{v_{id}}{2} - v_e \right) R$$

$$v_{od} = v_{o1} - v_{o2} = -g_{m1} \left(\frac{v_{id}}{2} - v_e \right) R + g_{m2} \left(-\frac{v_{id}}{2} - v_e \right) R$$

$$v_{od} = -\left(\frac{v_{id}}{2} - v_e \right) \left(g_m + \frac{\Delta g_m}{2} \right) R + \left(-\frac{v_{id}}{2} - v_e \right) \left(g_m - \frac{\Delta g_m}{2} \right) R = -g_m R v_{id} + \Delta g_m R v_e = -g_m R \left(v_{id} + \frac{\Delta g_m}{g_m} v_e \right)$$

At the emitter node :

$$\text{KCL at emitter node} \rightarrow i_{e1} + i_{e2} - i_e = 0 \rightarrow \left(\frac{v_{id}}{2} - v_e \right) \left(g_m + \frac{\Delta g_m}{2} + g_\pi \right) + \left(-\frac{v_{id}}{2} - v_e \right) \left(g_m - \frac{\Delta g_m}{2} + g_\pi \right) - G_{EE} v_e = 0 \quad \left(G_{EE} = \frac{1}{R_{EE}} \text{ \& } g_\pi = \frac{1}{r_\pi} \right)$$

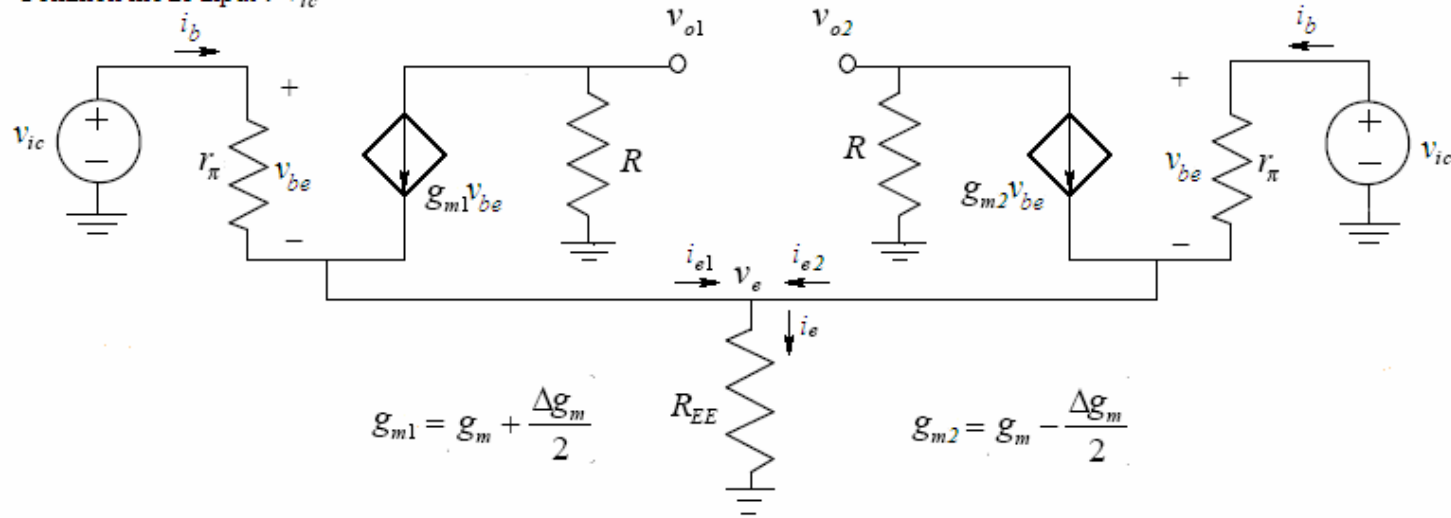
$$v_e = \frac{1}{2} \frac{\Delta g_m}{g_m} \frac{\beta_o R_{EE}}{r_\pi + (\beta_o + 1) 2 R_{EE}} v_{id} \cong \frac{1}{4} \frac{\Delta g_m}{g_m} v_{id} \quad \left(r_\pi \ll (\beta_o + 1) 2 R_{EE} \right)$$

$$v_{od} \cong -g_m R v_{id} \left(1 + \frac{1}{4} \frac{\Delta g_m^2}{g_m^2} \right) \quad \left| \quad A_{dd} = \frac{v_{od}}{v_{id}} \cong -g_m R = -300 \right.$$

negligible because
very low value

15.64 Continue

Common mode input : v_{ic}



There is no virtual ground since the circuit is mismatched

$$v_{be} = (v_{ic} - v_e)$$

$$v_{be} = (v_{ic} - v_e)$$

$$g_{m1} v_{be} = g_{m1} (v_{ic} - v_e)$$

$$g_{m2} v_{be} = g_{m2} (v_{ic} - v_e)$$

$$v_{o1} = -g_{m1} (v_{ic} - v_e) R$$

$$v_{o2} = -g_{m2} (v_{ic} - v_e) R$$

$$v_{od} = v_{o1} - v_{o2} = -g_{m1} (v_{ic} - v_e) R + g_{m2} (v_{ic} - v_e) R$$

$$v_{od} = -(v_{ic} - v_e) \left(g_m + \frac{\Delta g_m}{2} \right) R + (v_{ic} - v_e) \left(g_m - \frac{\Delta g_m}{2} \right) R = -\Delta g_m R (v_{ic} - v_e) = -\frac{\Delta g_m}{g_m} g_m R (v_{ic} - v_e)$$

At the emitter node :

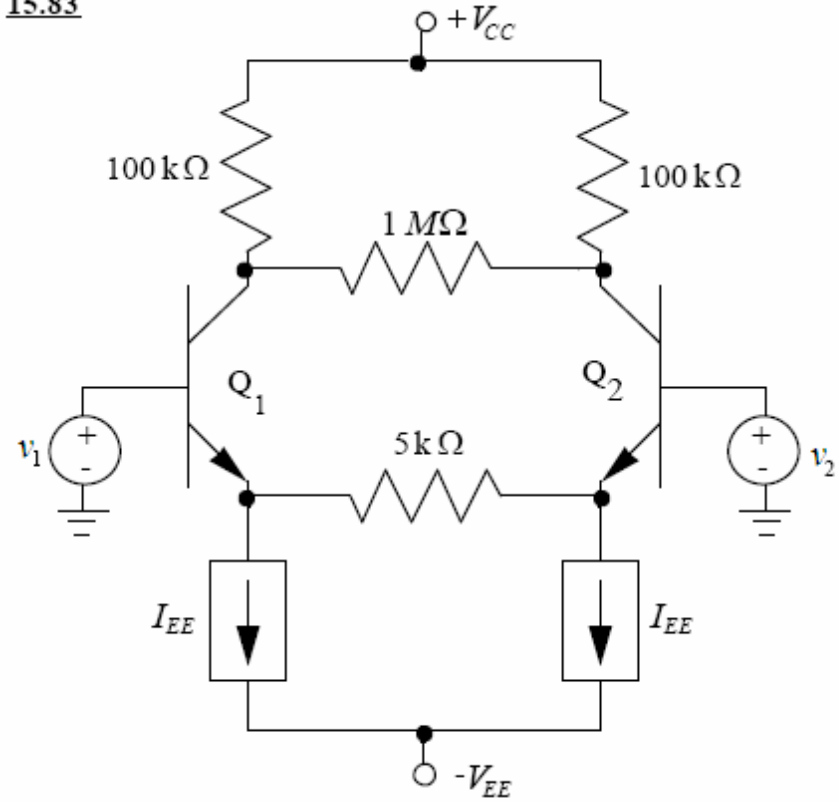
$$\text{KCL at emitter node} \rightarrow i_{e1} + i_{e2} - i_e = 0 \rightarrow (v_{ic} - v_e) \left(g_m + \frac{\Delta g_m}{2} + g_\pi + g_m - \frac{\Delta g_m}{2} + g_\pi \right) - G_{EE} v_e = 0$$

$$v_e = \frac{(\beta_o + 1) 2R_{EE}}{r_\pi + (\beta_o + 1) 2R_{EE}} v_{ic} \quad | \quad v_{ic} - v_e = \frac{r_\pi}{r_\pi + (\beta_o + 1) 2R_{EE}} v_{ic}$$

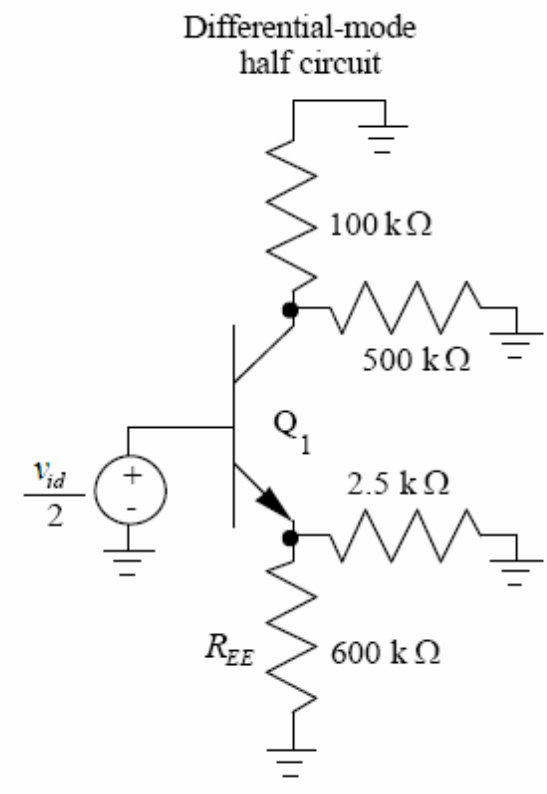
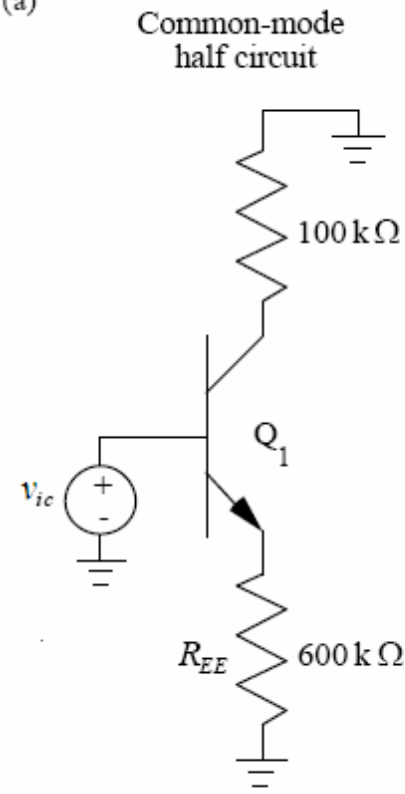
$$A_{cd} = \frac{v_{od}}{v_{ic}} = -\frac{\Delta g_m}{g_m} \frac{\beta_o R}{r_\pi + (\beta_o + 1) 2R_{EE}} \cong -\frac{\Delta g_m}{g_m} \frac{g_m R}{1 + 2g_m R_{EE}} = -0.00499$$

$$CMRR = \frac{A_{dd}}{A_{cd}} = (1 + 2g_m R_{EE}) \left[\frac{\Delta g_m}{g_m} \right]^{-1} = 60000 \text{ or } 95.6 \text{ dB}$$

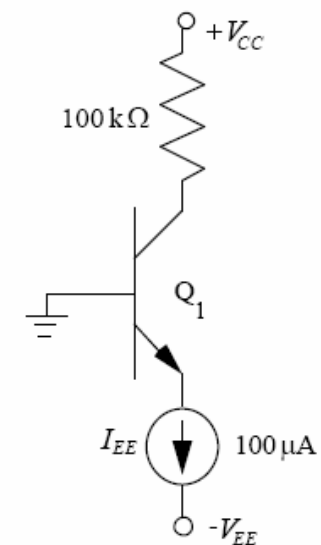
15.83



(a)



(b) dc half-circuit



Assume BJT Operates in Forward Active Region ;

$$I_C = \alpha_F I_E = \frac{100}{101} (100 \mu A) = 99.0 \mu A$$

$$V_{CE} = 20 - 10^5 I_C - (-0.7) = 10.8 V$$

$$V_{BC} = 10^5 I_C - 20 = -10.1 V$$

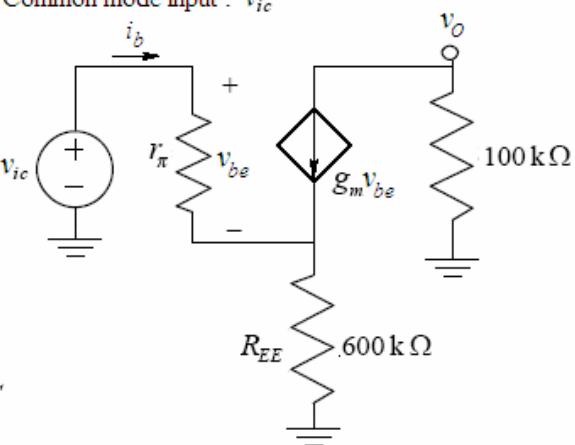
Assumption is Correct

$$Q - Point = (99.0 \mu A, 10.8 V)$$

$$r_\pi = \frac{100(0.025 V)}{99.0 \mu A} = 25.3 k\Omega$$

$$g_m = 40 I_C = 3.96 mS$$

Common mode input : v_{ic}



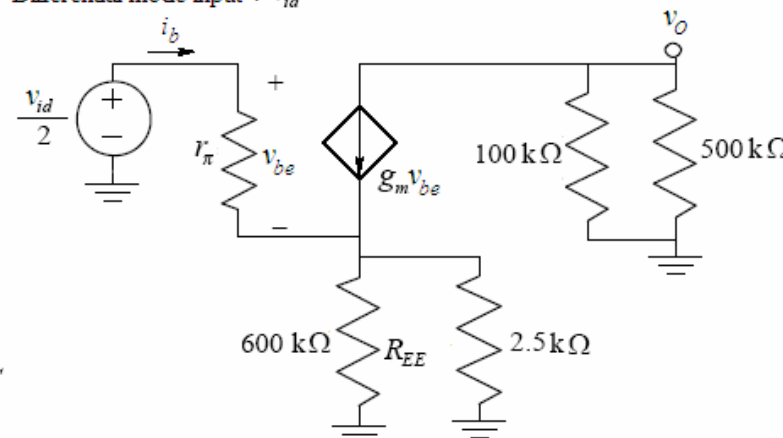
$$v_O = -g_m v_{be} (100 k\Omega)$$

$$v_{ic} = v_{be} + \left(g_m v_{be} + \frac{v_{be}}{r_\pi} \right) (600 k\Omega)$$

$$A_{cc} = \frac{v_O}{v_{ic}} = -\frac{\beta_o (100 k\Omega)}{r_\pi + (\beta_o + 1) (600 k\Omega)}$$

$$A_{cc} = -\frac{100 (100 k\Omega)}{25.3 k\Omega + 101 (600 k\Omega)} = -0.165$$

Differential mode input : v_{id}

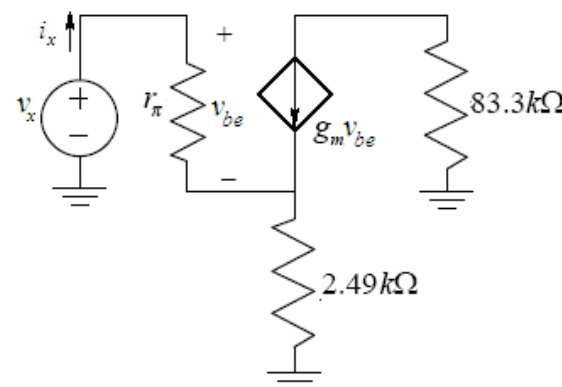


$$v_O = -g_m v_{be} (100 k\Omega \parallel 500 k\Omega)$$

$$\frac{v_{id}}{2} = v_{be} + \left(g_m v_{be} + \frac{v_{be}}{r_\pi} \right) (600 k\Omega \parallel 2.5 k\Omega)$$

$$A_{dd} = 2 \frac{v_O}{v_{id}} = -\frac{\beta_o (100 k\Omega \parallel 500 k\Omega)}{r_\pi + (\beta_o + 1) (600 k\Omega \parallel 2.5 k\Omega)}$$

$$A_{dd} = -\frac{100 (83.3 k\Omega)}{25.3 k\Omega + 101 (2.49 k\Omega)} = -30.1$$



$$R_{id} = 2 \frac{v_x}{i_x} = 2 [r_\pi + (\beta_o + 1) (2.49 k\Omega)]$$

$$R_{id} = 2 [25.3 k\Omega + 101 (2.49 k\Omega)] = 554 k\Omega$$

$$\text{Single - ended CMRR} = \left(\frac{0.5 A_{dd}}{A_{cc}} \right) = \left(\frac{15.05}{0.165} \right) = 91.2$$